EXTINCTION COEFFICIENTS FOR MULTIMODAL
ATMOSPHERIC PARTICLE SIZE DISTRIBUTIONS

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Abstract—A chart was developed which gives the volume-normalized extinction coefficient for log-normally modelled, multimodal particle volume distributions. The real part of the particle refractive index ranges from 1.30 to 1.70. Two adjustment charts modify the main chart for an imaginary part of the refractive index different from 0.02, and for a geometric standard deviation of the modal volume distribution, different from 2.0. The charts show that particles in the accumulation range, which has a mode in the 0.1 to 1 μm diameter size interval, are optically the most active. Exact knowledge of the particle refractive index is imperative for fine particles, but of small importance for coarse particles.

INTRODUCTION

The extinction coefficient of a real atmosphere, when used in the Lambert–Beer law, defines the change in intensity of light traversing a given path length (e.g., Waggoner and Charlson, 1976). Given a minimum contrast ratio for an object against the horizon brightness, the extinction coefficient is used to calculate the visible range (Middleton, 1968). Light attenuation measurements and calculations enter global energy balance estimations (e.g., Braslau and Dave, 1973; Liou and Sasamori, 1975; Temkin et al., 1975) and response predictions for optical measurement devices such as nephelometers, transmissometers, and photometers (e.g., Beutner, 1974; Charlson et al., 1968; Quenzel et al., 1975).

The extinction coefficient is given by the extinction properties of the atmospheric gases and particles. A further distinction is made between the incident light absorbed and scattered by the particles. In polluted air environments, light extinction by particles usually exceeds that by vapors and depends on the size distribution, concentration and nature of the airborne particles. An experimentally determined size distribution or a model size distribution, based on measurements, therefore enters all calculations of reflected and transmitted radiance of the earth’s atmosphere.

Plass and Katawar (1968) and Liou and Sasamori (1975) used a power law function for the atmospheric particle number size distribution (Junge, 1963). Weinman et al. (1975) used such a distribution for multiply scattered sunlight. Davies (1976a, b) showed that Junge power law distributions which have been used for sizes larger than about 0.1 μm in diameter, do not correctly represent the size distributions of atmospheric aerosols. Deirmendjian (1969) applied modified
gamma functions to the aerosol number concentrations, which were used by Braslau and Dave (1975) and Dave et al. (1975) in their calculations of the spectral distribution of transmitted solar energy.

Recent experimental investigations have shown that atmospheric size distributions are multimodally distributed, and that representations of the linear mass or volume density functions, divided by the logarithm of the particle size and plotted against the logarithm of particle size, give more detail of the size distribution and are, therefore, preferred for the study of particle formation and transformation (Berry, 1967; Whitby et al., 1972; Willeke and Whitby, 1975).

This work takes as a basis such volumetric size distributions, and describes charts which have been developed for the prediction of light extinction in the presence of multimodal particle volume distributions of varying median size, and geometric standard deviation with varying real and imaginary components of the particle refractive index. In earlier studies, Fontzik (1965) and Bergstrom (1973) calculated the extinction coefficients of atmospheric aerosols for log-normally distributed particle number distributions. Ensor and Pilat (1971) applied a similar technique to source aerosols coming from stacks. Davies (1975) calculated the visual range for particles of several refractive indices.

DEVELOPMENT OF CHARTS

The light energy dissipated by a spherical particle per energy incident per unit cross-sectional area of the sphere, called the extinction cross section, \( \sigma_{\text{ext}} \), was calculated from Mie’s (1908) theory by use of a computer program which was written by Dave (1968), modified by Ensor (1972), and further modified by Sverdrup (1976). In conventional presentations of the optical properties of aerosols, the results are plotted as the extinction efficiency factor, \( Q_{\text{ext}} \), vs. the size...
Fig. 1. Dependence of volume-normalized single particle cross section on particle size.

parameter $\alpha = \pi D/\lambda$, where the extinction efficiency factor is the extinction cross section divided by the geometric cross-sectional area of the spherical particle, $D$ is the particle diameter, and $\lambda$ is the wavelength of the illuminating light. We deviate from this convention by dividing the extinction cross section by the particle volume, $V$, so that we may integrate this curve with the particle volume distribution more readily.

For example, Fig. 1 gives the specific extinction, scattering and absorption cross sections in a product with wavelength $\lambda$ as a function of $D/\lambda$ for an aerosol with particle refractive index $m = 1.5+0.02i$. One may conclude from the non-dimensional plot of Fig. 1 that aerosols are optically most active at particle diameters of about 0.5 $\mu$m for a mean visible wavelength of $\lambda = 0.55 \mu$m. Figure 1 also shows that particles smaller than about 0.05 $\mu$m or larger than about 5 $\mu$m contribute very little to light attenuation.

In Fig. 1, the sum of absorption cross section $\sigma_{abs}$ and scattering cross section $\sigma_{sca}$ gives the extinction cross section $\sigma_{ext}$. In this paper, we confine our discussions to the development and use of extinction coefficient charts. Further details, including curves on particle scatter and absorption, are given by Brockmann (1976).

Given particle concentration distribution $dN/dD$, the extinction coefficient $\beta_{ext}$ can be calculated from

$$\beta_{ext} = \int_{0}^{\infty} \frac{dN}{dD} dD. \quad (1)$$

For a volume distribution on a linear-log scale (Berry, 1967; Whitby et al., 1972; Willeke and Whitby, 1975), we have

$$\beta_{ext} = \int_{-\infty}^{\infty} \frac{\sigma_{ext}}{VT_i} \frac{dV}{d\log(D)} d\log(D/\lambda). \quad (3)$$

For a multimodal volume distribution, we may take the total volume under each mode $i$, $VT_i$, and define a volume-normalized extinction coefficient $\beta_{ext}'$,

$$\beta_{ext}' = \frac{\int_{-\infty}^{\infty} \frac{\sigma_{ext}}{VT_i} \frac{dV}{d\log(D/\lambda)} d\log(D/\lambda)}{VT_i}. \quad (4)$$

Whitby (1975) showed that the individual modes in a multimodal particle volume distribution may be represented by the log-normal volume distribution

$$\frac{dV}{d\log(D)_{i}} = \frac{VT_i}{\sqrt{2\pi \text{log } \sigma_g^2}} \exp \left\{ -\frac{\text{log } D_{i}^2}{2\text{log } \sigma_g^2} \right\}. \quad (5)$$

where $\sigma_g = $ geometric standard deviation and $D_i = $ median particle diameter of mode $i$.

Integration of Equations (3) and (4) with the volume distribution representation of Equation (5) gives the volume-normalized extinction coefficient, plotted in Fig. 2 as a function of median particle size $D_i$. The geometric standard deviation was fixed at $\sigma_g = 2.0$, which was found to be a representative aver-
Extinction coefficients for atmospheric particle size distributions

Fig. 3. Adjustment factors for particle volume distributions with an imaginary part of the particle refractive index equal to 0.06.

A further adjustment factor is given in Fig. 4 for the case when the geometric standard deviation differs from the average value of \( \sigma_g = 2.0 \). By use of Fig. 4, one may interpolate for \( \sigma_g \) values within the range 1.6 \( \leq \sigma_g \leq 3.5 \), which covers most modes found in atmospheric environments.

A geometric standard deviation of unity would represent a monodisperse aerosol and would result in an extinction curve of the same shape as shown in Fig. 1. When the specific cross section, as seen in Fig. 1, is integrated with a particle volume distribution curve having a high \( \sigma_g \), the wiggles in Fig. 1 are integrated out, the peak amplitude is depressed, and the resulting curve itself will have a wide spread. The \( \sigma_g \) adjustment curves of Fig. 4 show that a high \( \sigma_g \) of the volume distribution depresses the peaks in Fig. 2 and increases the wings of the curves.

If both adjustment curves are used simultaneously, the actual extinction coefficient may differ from the value found through use of the charts. However, the error exceeds 5% only for large or very small particle sizes, where the absolute value of the optical extinction coefficient is very small (Brockmann, 1976). The error may therefore be considered negligible for most atmospheric particles of interest. The assumption of particle sphericity is likely to give a higher uncertainty than any error incurred by use of the charts.

**SAMPLE CALCULATION**

The use of the charts is best illustrated through an example. Figure 5 shows the volume distribution

![Graph](image-url)
Assuming a low particle refractive index (e.g., $m = 1.33$ for a water droplet),

$$\beta_{\text{ext, aerosol}} = 1.16 \times 10^{-4} \text{ [}/\text{m}] \text{ for } m = 1.30-0.00i.$$  (13)

Assuming a high particle refractive index,

$$\beta_{\text{ext, aerosol}} = 3.10 \times 10^{-4} \text{ [}/\text{m}] \text{ for } m = 1.70-0.06i.$$  (14)

Equations (13) and (14) show that an atmospheric aerosol distribution of hygroscopic particles of low refractive index results in a visual range 2.5 times greater than the same aerosol size distribution would give with particles of high refractive index.

The exact knowledge of the particle refractive index is therefore of utmost importance. The effect of humidity on particle refractive index for a given size distribution may, for example, be calculated by Hänel's (1971) or Tuomi's (1975) technique. The present charts permit the use of a different mean refractive index for each mode. Note also from Fig. 2 that knowledge of the refractive index is only important for fine particles, not for coarse particles with a mean particle diameter larger than about 2 $\mu$m ($\lambda = 0.55$ $\mu$m). Figure 2 also shows that high-refractive-index particles in the ambient atmosphere have their peak optical activity at about half the particle size of low-refractive-index particles.

The extinction coefficient for the atmospheric gas environment is strongly wavelength-dependent (Middleton, 1968). For mean wavelength $\lambda = 0.55$ $\mu$m, we find from Fig. 2 that the volume-normalized extinction coefficient due to particles is for the accumulation range mode (mode 1)

$$\beta_{\text{ext, aerosol}} = 5.5 \times 10^{-6} \text{ [}(1/\text{m})(\mu\text{m}^3/\text{cm}^3)]$$

for $\lambda = 0.55$ $\mu$m, $\sigma_g = 2.0$.  (8)

The adjustment for $\sigma_g = 2.1$ is, from Fig. 4,

$$\beta_{\text{ext, aerosol}} = 0.98.$$  (9)

The extinction coefficient for mode 1 is therefore

$$\beta_{\text{ext, aerosol}} = \beta_{\text{ext, aerosol}}(V) = 1.74 \times 10^{-4} \text{ [}/\text{m}].$$  (10)

Similarly, the extinction coefficient for mode 2 is

$$\beta_{\text{ext, aerosol}} = 0.37 \times 10^{-4} \text{ [}/\text{m}].$$  (11)

Note that the light extinction due to the coarse particle mode is only about 1/5 of the extinction due to the accumulation mode, although the majority of the particle volume is in the coarse particle mode. The main chart, Fig. 2, shows clearly that particles in the accumulation range, from about 0.1 to 1.0 $\mu$m, are optically the most active when irradiated by visible light.

The extinction coefficient due to the entire particle volume distribution is the sum of Equations (10) and (11),

$$\beta_{\text{ext, aerosol}} = 2.11 \times 10^{-4} \text{ [}/\text{m}] \text{ for } m = 1.50-0.02i.$$  (12)

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high humidity of 79% in the example gives a visual range of 12 km.

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REFERENCES


